The first value of a_1 is worthy of note; for if it were required to determine the position of four circles b, c, b_1, c_1 of given magnitude, that touch each other consecutively, so that a circle could be drawn touching all, the equation $a_1 = a$, shows that each of these circles fulfills this condition.

Again, a, b, c, being the radii of the first, we have the following relations connecting the radii of the circles of the various groups.

$$\left.\begin{aligned}\frac{1}{a_1}&=\frac{1}{m}-\frac{1}{a}\\ \frac{1}{b_1}&=\frac{1}{m}-\frac{1}{b}\\ \frac{1}{c_1}&=\frac{1}{m}-\frac{1}{c}\end{aligned}\right\}\text{2nd,}\quad \left.\begin{aligned}\frac{1}{a_2}&=\frac{1}{m_1}-\frac{1}{a_1}\\ \frac{1}{b_2}&=\frac{1}{m_1}-\frac{1}{b_1}\\ \frac{1}{c_2}&=\frac{1}{m_1}-\frac{1}{c_1}\end{aligned}\right\}\text{3rd,}\quad \left.\begin{aligned}\frac{1}{a_x}&=\frac{1}{m_{x-1}}-\frac{1}{a_{x-1}}\\ \frac{1}{b_x}&=\frac{1}{m_{x-1}}-\frac{1}{b_{x-1}}\\ \frac{1}{c_x}&=\frac{1}{m_{x-1}}-\frac{1}{c_{x-1}}\end{aligned}\right\}x+1.$$

Now substituting for $1 \div a_1$, $1 \div b_1$, $1 \div c_1$ in the third group, their values as given in the second, and carrying the resulting values of $1 \div a_2$, $1 \div b_2$, $1 \div c_2$ into the fourth, and so on, to the $(x+1)$th group, we find that

$$\frac{1}{a_x}+\frac{1}{a}=\frac{1}{b_x}+\frac{1}{b}=\frac{1}{c_x}+\frac{1}{c}=\frac{1}{m_{x-1}}-\frac{1}{m_{x-2}}+\ldots+\frac{1}{m},$$

when x is odd, and

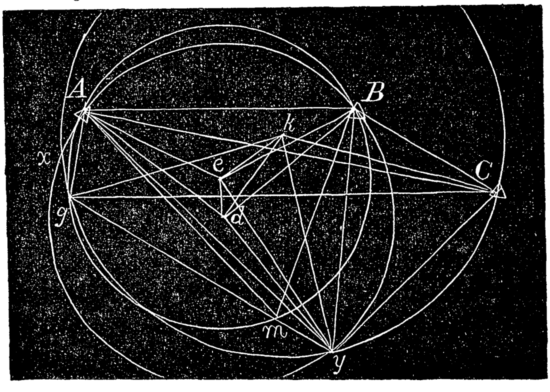
$$\frac{1}{a_x}-\frac{1}{a}=\frac{1}{b_x}-\frac{1}{b}=\frac{1}{c_x}-\frac{1}{c}=\frac{1}{m_{x-1}}-\frac{1}{m_{x-2}}+\ldots-\frac{1}{m}$$

when x is an even number.

A PROBLEM IN SURVEYING.

BY T. J. LOWRY, M. S., SAN FRANCISCO, CALIFORNIA.

Problem:—Required the positions of the two places of observation y and m, with reference to three known points A, B and C, having observed at m the angles AmB and Bmy and at y the angles ByA and Cym.



Trig. Analysis:—In the isosceles $\triangle$ ABe we have the base AB and $\angle$ AeB ($=2AmB$), and hence all the $\angle$s to find Ae or Be. And in $\triangle$ Ade are known $\angle Aed$ ($=180° - AmB$), side Ae, and $\angle Ade$ ($=AyB$) to get Ad and de. Now in isosceles $\triangle$ Age having sides Ae and ge, and $\angle$ Aeg [=

$2(180° — AmB — Bmy)$] we get Ag. Then in $\triangle\ gAC$ are given gA, AC, and $\angle gAC\ (= gAe + eAB — BAC)$ to find gC and $\angle AgC$. And then in isosceles $\triangle\ gkC$ we have gC and $\angle\ gkC\ [= 2(180° — gyC)]$ and hence all the angles, to determine $gk\ (= kC)$. Now in $\triangle\ kge$ are known kg, ge and $\angle kge\ (= kgC + Age — AgC)$ to get ke and $< gek$.

In $\triangle\ ked$ we know de, ek and $< dek\ (= 360° — gek — ged)$ and hence all the angles and side dk. And in $\triangle\ dky$ we have all the sides and hence all the angles. In $\triangle\ yke$ are known ke, ky and $< eky\ (= dky + dke)$ to find ey. Then in $\triangle\ Aey$ having ey, Ae and $< Aey\ (= 360° — yek — gek + Aeg)$ we find Ay. And in $\triangle\ ABy$ are known Ay, AB and observed $< AyB$ to get yB and $< yAB$. Now in $\triangle\ CAy$ we have Ay, AC and $< yAC\ (= yAB — BAC)$ to find $< AyC$. Then since $< Aym = Cym — AyC$, and the $< yAm = 180° — Bmy — BmA — Aym$, we have in the $\triangle\ mAB$ known the $< mAB\ (= yAB + yAm)$, the side AB, and the $< AmB$ to find Am and Bm.

Geom. Const.:—Through A and B lay down circles of position containing respectively the angles AmB and AyB. Then from AB at A lay off $< BAg = Bmy$ and at B the $< ABg = 180° — AmB — Bmy$, and g the point of intersection of the two lines thus drawn will be a secant point of the circle ABm and the right line through m and y. Now through g and C sweep a circle of position containing the observed angle gyC, and y, a secand point of this and the position circle ABy, is one of the required places of observation (an approximate knowledge of his position will in general tell the observer whether he was at y or x); then draw the right line yg and m the point where it cuts the circle ABm is the other place of observation.

A more general statement of the Rule given at page 146, vol. I, of the ANALYST, for plotting the centre of a circle of position is: — At each end of the line, joining the two observed signals, lay off the difference between the observed angle and 90°, on the same side of this line as the place of observation if the observed angle is less than 90°, but if greater than 90°, on the opposite side. And if the observed angle $= 90°$ then the centre of the position circle is at the middle point of the line joining the signals.

SOLUTION OF PROBLEM 94. (*SEE PAGE* 199, *VOL. II.*)

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FROM the figure we easily obtain,

$$\cos a = \frac{y^2 + (c + x)^2 + (a + s)^2 — r^2}{2(a + s)\sqrt{[y^2 + (c + x)^2]}}, \quad \ldots\ldots\ldots (1)$$